CIVIL-312: Hydraulic Engineering and Infrastructures

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Pressurized Pipe Flow

Information

Tentative Assignments Schedule:

- **HW 1**: Assigned in **Week 5** → Due in **Week 8**
- **HW 2**: Assigned in **Week 9** → Due in **Week 12**
- **HW 3**: Assigned in **Week 13** → Due in **Week 15*** (Note: due the week after the last week of class, before the break)

Although it should be close to this schedule, please keep in mind that it might change slightly depending on how the course is progressing

Topics for this week

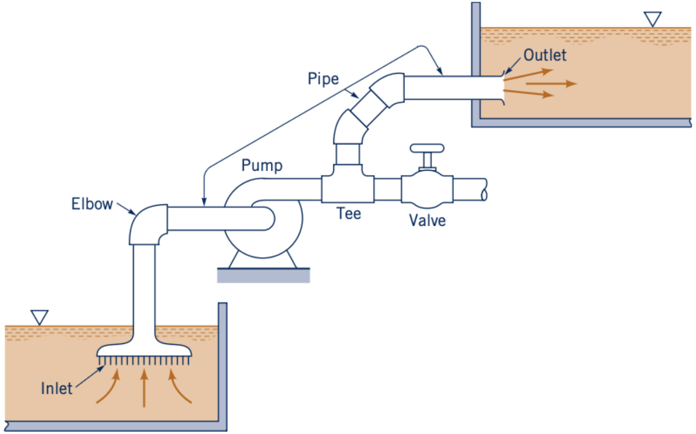
- Intro on pipes & *laminar* vs *turbulent* flows
- Energy balance
- Stress & velocity distribution, and headlosses
- Major and minor losses

Common features of *closed conduits*

- We use *close conduits* (commonly called a *pipe* if it is of round cross section or a *duct* if it is not round) to transport fluids (liquid of gas)

- Examples are everywhere in our industrialized society:
  - Water distribution and waste water collection urban systems (civil & environmental engineering!)
  - Transport of chemicals and oil for industrial and energy applications
  - Air distribution and conditioning in buildings
  - Etc.

- Pipe systems are made of components:
  - pipes themselves (perhaps of more than one diameter)
  - Fittings to connect individual pipes (e.g., elbows)
  - Flow rate control devices (valves)
  - Pumps and turbines to add or extract energy from the system



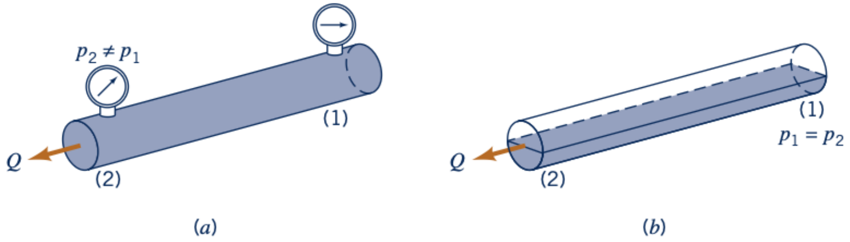
General characteristics of pipe flow

- Although not all of the conduits that transport fluids are round in cross-section, most of them are → water pipes, penstocks, hydraulic hoses, etc.

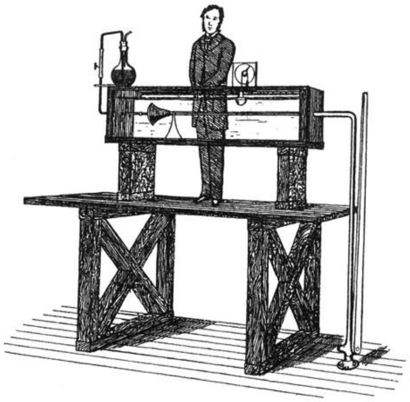
- Designed to withstand a considerable pressure difference across their walls without undue distortion of their shape.

- *Pipe Flow* typically refers to *pressurized flow*: the pipe is completely filled with the fluid being transported and pressure is the main mechanism that drives the flow (figure a)

- AS OPPOSED TO *open channel flows* where the main driver of the flow is gravity alone and there is a free surface (figure b).



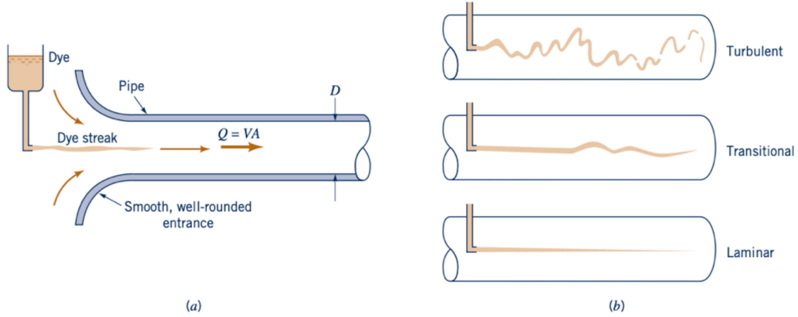
General characteristics of pipe flow



- Flow In a pipe may be ***laminar* or *turbulent***. Turbulent flows are chaotic, caused by eddies of various size that keep mixing the particles. Laminar flow are smooth (not much mixing)

- Osborne Reynolds (1842–1912) was the first to show this classification with its famous red dye experiment

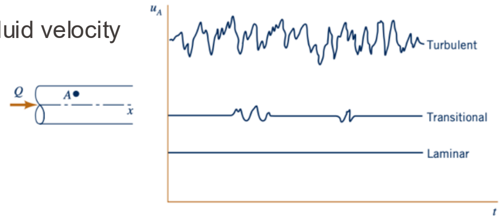
    - For ”small enough flowrate” the dye streak remain well defined (laminar); for ”intermediate flowrate” the dye fluctuate in space and time (intermediate); for “large enough flowrates” the dye become blurred (turbulent).



How small is small and how large is large?

The fluid viscosity is what create resistance to the flow; higher flows can overcome the resistance more easily

Time dependence of fluid velocity at a point.



The ratio between the inertial forces (the flow) and the viscous forces is then what determines "small" and "large" and therefore what is *laminar* and what is *turbulent*!

$$\mathcal{R}e = \frac{Inertial\ Forces\ (\sum F\_ext)}{Viscous\ Forces}$$

$$\sum F_{ext} = ma = [\rho \mathbb{V} a] \sim \rho \frac{l^3 V^2}{l} \sim \rho l^2 V^2$$

l: length; V: velocity; ρ: density; μ: dynamic viscosity

$$F_{visc} = \tau A = \left[\mu \frac{dV}{dy} A\right] \sim \mu \frac{V}{l} l^2 \sim \mu V l$$

$$\mathcal{R}e = \frac{\rho l^2 V^2}{\mu V l} = \rho \frac{lV}{\mu} = \frac{lV}{\nu}$$

where ν is the kinematic viscosity ($\nu = \rho/\mu$)

The Reynolds Number ($\mathcal{R}e$)

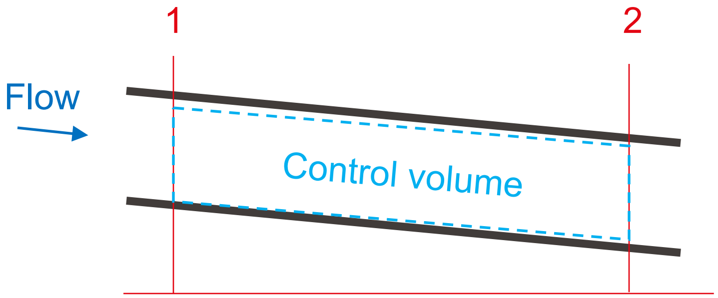
The Reynolds number is one of the most important dimensionless parameter for pipe flows.

$$\mathcal{R}e = \rho V D/\mu$$

where V is the average velocity in the pipe and D is typically the diameter

EPFL

Energy Balance in pipes



Let's apply the Energy Balance equation we derived during week 1:

$$\frac{dQ_H}{dt} - \frac{dW_S}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\left(e_u + \frac{V^2}{2} + gz\right) dA + \int_{CS}\left(\frac{P}{\rho} + e_u + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot\hat{n}\,dA$$

0

We apply energy balance between cross-sections 1 and 2 for a **steady flow** ($\partial/\partial t = 0$):

Flow enters perpendicularly in pipes so $\vec{V}\cdot\hat{n} = V$

$$\frac{dQ_H}{dt} - \frac{dW_S}{dt} = \int_{A_2}\left(\frac{P_2}{\rho} + e_{u_2} + \frac{V_2^2}{2} + gz_2\right)\rho V_2\,dA_2 - \int_{A_1}\left(\frac{P_1}{\rho} + e_{u_1} + \frac{V_1^2}{2} + gz_1\right)\rho V_1\,dA_1$$

$$\frac{dQ_H}{dt} - \frac{dW_S}{dt} = \int_{A_2}\left(\frac{P_2}{\rho} + e_{u_2} + gz_2\right)\rho V_2\,dA_2 + \int_{A_2}\rho\frac{V_2^3}{2}dA_2 - \int_{A_1}\left(\frac{P_1}{\rho} + e_{u_1} + \frac{V_1^2}{2} + gz_1\right)\rho V_1\,dA_1 - \int_{A_2}\rho\frac{V_1^3}{2}dA_1$$

We assume that the flow is **uniform flow**, thus:

1) Hydrostatic conditions prevail at CS 1 and 2 → the term $\frac{P}{\rho} + e_u + gz$ is constant across CS 1 and 2 and can be brought out of the integral

2) the $\int_A \rho V\, dA = \dot{m} \implies \int_A \rho \frac{V^3}{2}\, dA = \dot{m}\frac{V^2}{2}$ and by **continuity**, $\dot{m}_1 = \dot{m}_2 = \dot{m}$

Hence:

$$\frac{dQ_H}{dt} - \frac{dW_S}{dt} = \left(\frac{P_2}{\rho} + e_{u_2} + gz_2\right)\dot{m} + \dot{m}\frac{V_2^2}{2} - \left(\frac{P_1}{\rho} + e_{u_1} + gz_1\right)\dot{m} - \dot{m}\frac{V_1^2}{2}$$

Dividing everything by $\dot{m}g$:

$$\left(\frac{1}{\dot{m}g}\right)\frac{dQ_H}{dt} - \left(\frac{1}{\dot{m}g}\right)\frac{dW_S}{dt} = \left(\frac{P_2}{\gamma} + \frac{e_{u_2}}{g} + z_2\right) + \frac{V_2^2}{2g} - \left(\frac{P_1}{\gamma} + \frac{e_{u_1}}{g} + z_1\right) - \frac{V_1^2}{2g}$$

The shaft work term dW_s/dt can be the result of:

- A *turbine* $W_T \Rightarrow$ work of the fluid done one the surrounding $\Rightarrow$ OUT (+)
- A *pump* $W_P \Rightarrow$ work on the fluid system from outside $\Rightarrow$ IN (-)

$$\frac{dW_S}{dt} = \frac{dW_T}{dt} - \frac{dW_P}{dt}$$

Note that each term has dimension of a LENGTH!

Hence

$$\left(\frac{1}{\dot{m}g}\right)\frac{dW_P}{dt} + \frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \left(\frac{1}{\dot{m}g}\right)\frac{dW_T}{dt} + \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \left[\left(\frac{e_{u_2} - e_{u_1}}{g}\right) - \left(\frac{1}{\dot{m}g}\right)\frac{dQ_H}{dt}\right]$$

Head supplied by Pumps h_P

Head "supplied" by Turbines h_T

HEADLOSS h_L

This is very important for designing pipe flow: most of the losses occur because of friction in/of the flow!!

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_P = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_T + h_L$$

Where:

$P/\gamma =$ Pressure head

$P/\gamma + z =$ Piezometric head

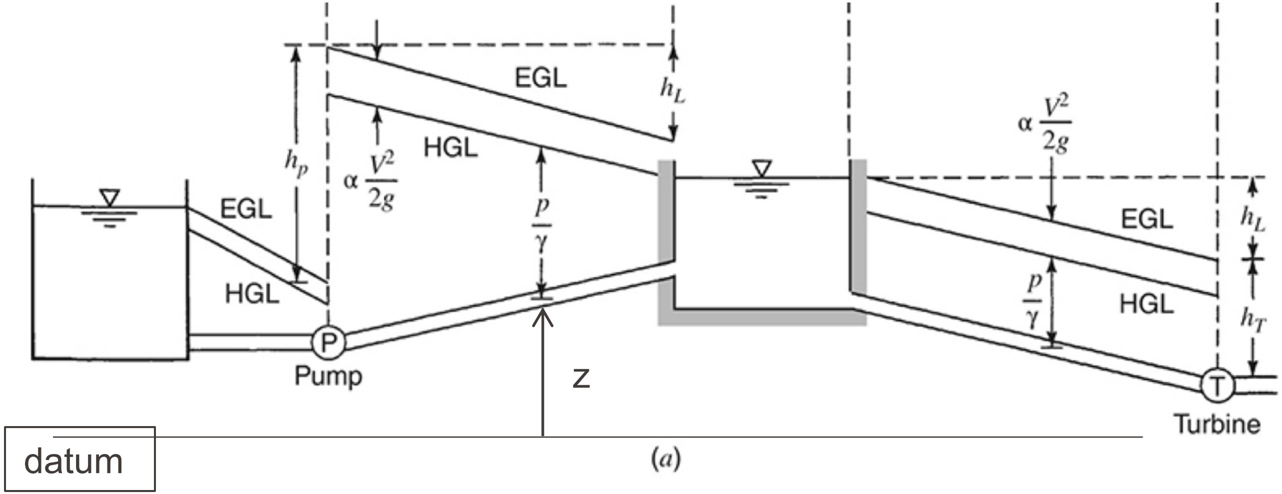
$V^2/2g =$ Velocity head

$$\frac{P}{\gamma} + z + \frac{V^2}{2g} = H$$

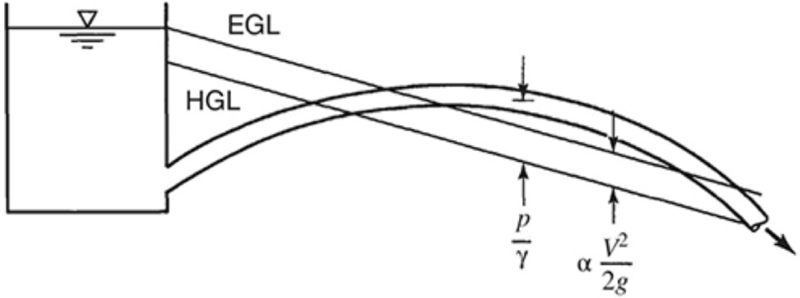
Total Head

*all these terms have dimension of length (units in meters)

Note: because the velocity is actually not uniform across the cross-section, we usually add the *kinematic energy factor* $\alpha = \frac{\int_A V^3 dA}{A\bar{V}}$ (see notes in Energy equation derivation form Week 1)



(a)

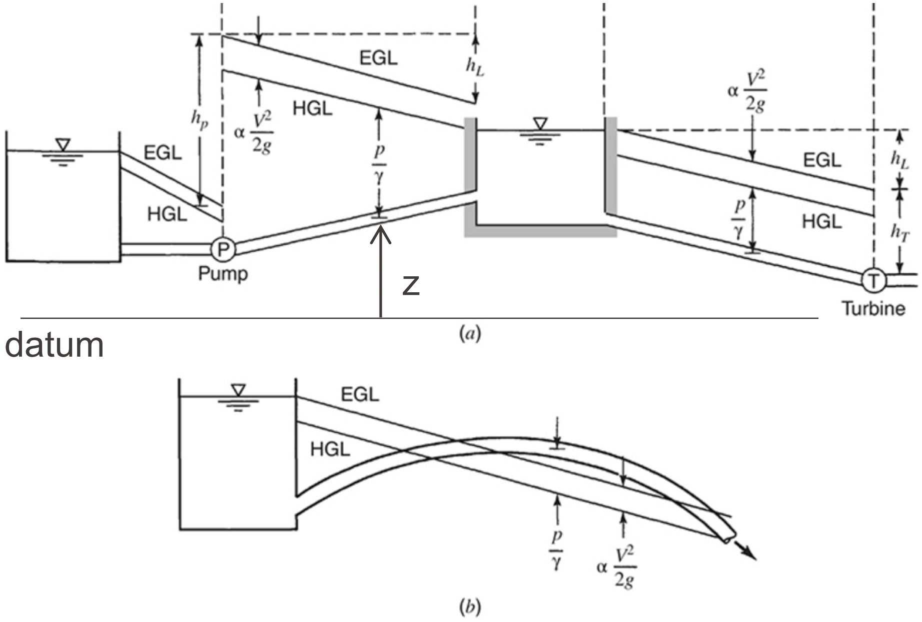


(b)

EGL = Energy Grade Line, it's the line showing the total head (aka, total energy) along the pipe.

HGL = Hydraulic Grade Line = just $\frac{P}{\gamma} + z$, i.e., the pressure head plus the elevation up to the center of the pipe.

$$HGL = EGL - \frac{V^2}{2g}$$



- Because of headlosses, the EGL will always slope downwards in the direction of flow except where a pump supplies (pressure) energy to the system. In this case there will be an abrupt rise in the EGL and HGL.
- A turbine takes energy out of a system and consequently the EGL and HGL will decrease abruptly after passing through a turbine.
- Where the pressure is zero, the HGL coincides with the system fluid.
- If the pipe diameter changes at some point in a system, the velocity of the flow will also change. Also, the distance between the EGL and HGL will change.
- If the HGL falls below the pipe it means that HGL< z and therefore p/γ is negative, indicating sub-atmospheric pressure.

Important note:

The headloss h_L *is the loss of mechanical energy due to **stresses** in the flow.*

Viscous Stresses

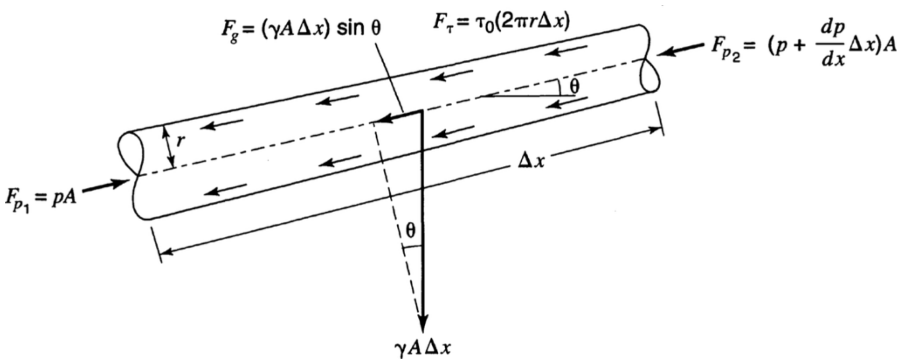
Stresses due to the viscosity of the fluid = the friction of fluid particles moving onto each other

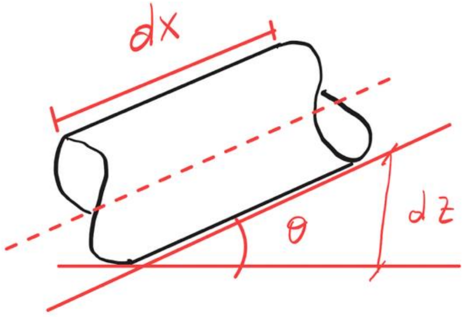
Turbulent Stresses

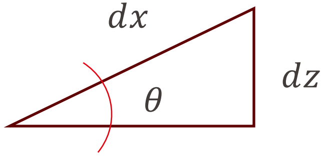
Stresses due to turbulence = macro particles of fluid that hit each other carried by eddies (vortices) and exchanging momentum

h_L are very important with it comes to designing/dimensioning pipes. We will find an expression for both laminar and turbulent flow. But first we need to look at what causes them: the stress.

Velocity and Stress Distribution, and Headlosses







$$\frac{dz}{dx} = \sin\theta$$

1) Let's apply the momentum equation on the cylindrical element of fluid in figure in the x-diection

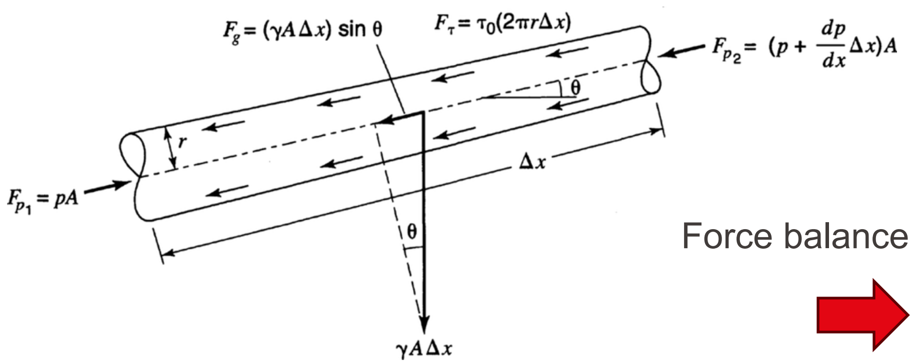
2) Consider steady flow (laminar or turbulent) → $\partial/\partial t = 0$

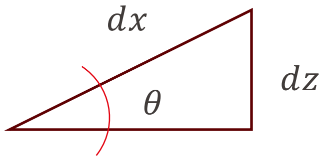
3) For a cylindrical element with no variation of cross-section, the velocity in is equal to the velocity out so the net flux of momentum in/out is zero

$$\sum F_x = \frac{\partial}{\partial t}\int_{C\mathbb{V}} \rho V_x d\mathbb{V} + \int_{CS} \rho \vec{V}\vec{V}\cdot\hat{n}dA$$

=0 because steady (on average)

=0 velocity is the same in and out (on average)





$$\frac{dz}{dx} = \sin\theta$$

Force balance on the small element

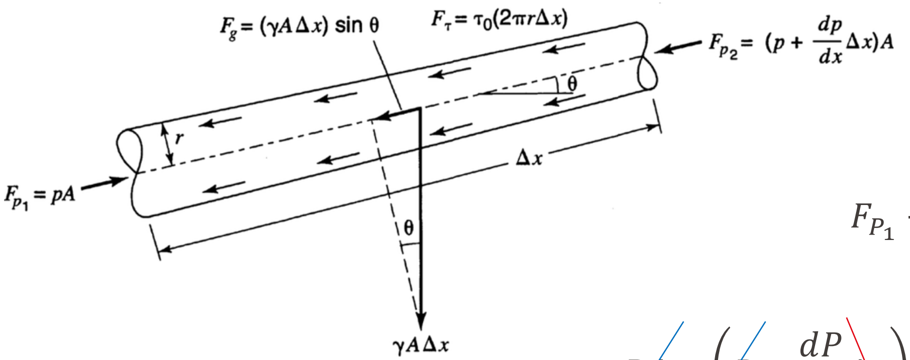
$$\sum F_x = 0$$

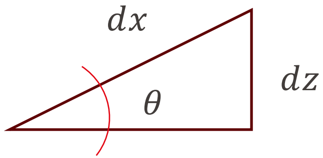
- Pressure forces: $F_{P_1} = PA$

$$F_{P_2} = \left(P + \frac{dP}{dx}\Delta x\right)A$$

- X-component of the gravity force: $F_g = (\gamma A \Delta x)\sin\theta$

- <u>Shear Stress</u> on surrounding surface $F_\tau = \tau(2\pi r \Delta x)$





$$\frac{dz}{dx} = \sin\theta$$

$$F_{P_1} - F_{P_2} - F_g - F_\tau = 0$$

$$PA - \left(P + \frac{dP}{dx}\Delta x\right)A - (\gamma A \Delta x)\sin\theta - \tau(2\pi r \Delta x) = 0$$

$$A = \pi r^2$$
$$\sin\theta = dz/dx$$

$$\frac{dP}{dx}(\pi r^2) - \gamma(\pi r^2)\frac{dz}{dx} - \tau(2\pi r) = 0$$

Dividing everything by r and rearranging

$$\tau = \frac{\gamma r}{2}\left(-\frac{d}{dx}\left(\frac{P}{\gamma} + z\right)\right)$$

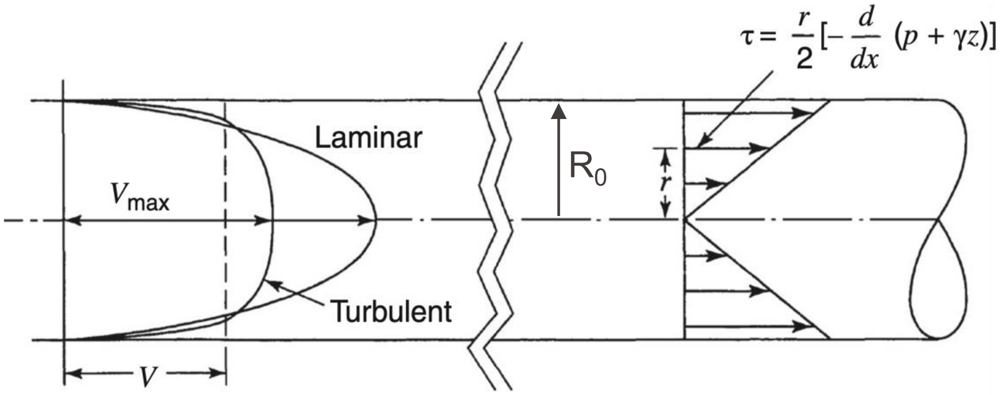
$$\tau = \frac{\gamma r}{2}\left(-\frac{d}{dx}\left(\frac{P}{\gamma} + z\right)\right)$$

$$i = -\frac{d}{dx}\left(\frac{P}{\gamma} + z\right)$$

i is the *driver of the flow*. It's the gradient that moves flow in pipes. It's constant and negative across the flow section for uniform flows

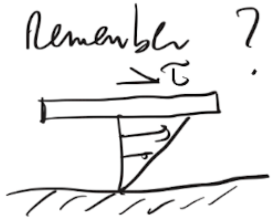
$$\tau = \gamma\, i \frac{r}{2}$$

- $\tau = 0$ in the center of the pipe (r=0)
- $\tau = \tau_{MAX}$ at the border (r=R_0)



- In LAMINAR Flows → τ is all due to *viscosity* (friction between moving particles)
- In TURBULENT Flows → τ will also be due to *turbulent stresses* (we'll see it later)

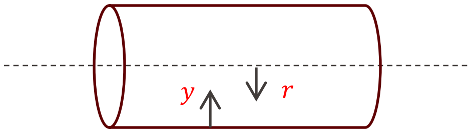
Laminar Flow ($Re <<<$)



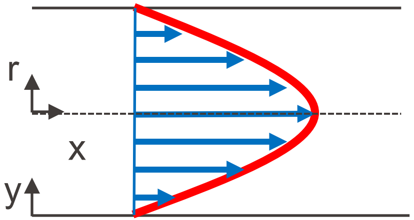
$$\tau = \mu \frac{dV}{dz}$$

In week 1 we defined viscosity as the measure of resistance of a fluid to shear

Change spatial variable y to r for pipe: $\frac{dV}{dy} = -\frac{dV}{dr}$



$$\tau = -\mu \frac{dV}{dr}$$



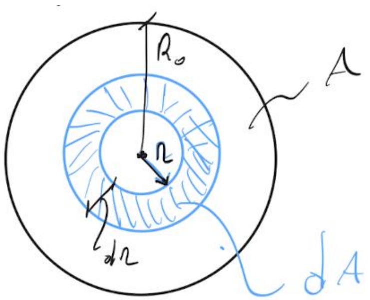
We now substitute the expression of the shear stress we just found and integrate for the whole radius of the pipe:

$$\gamma i \frac{r}{2} = -\mu \frac{dV}{dr} \longrightarrow \int_{V_x}^{0} -dV = \int_{r}^{R_0} \frac{\gamma}{\mu}\, i \frac{r}{2}\, dr \longrightarrow V_x = \frac{\gamma i}{4\mu}\left(R_0^2 - r^2\right)$$

Note that velocity is 0 at the pipe walls, so $r = R_0 \Rightarrow V = 0$

Parabolic distribution

How much is the discharge Q in a pipe in laminar flow?



$$dA = 2\pi r\, dr$$

$$Q = \int_A V_x dA = \int_r V_x 2\pi r\, dr$$

$$= \int_r^{R_0} \frac{\gamma i}{4\mu}(R_0^2 - r^2)\, 2\pi r\, dr$$

$$= \frac{\gamma i \pi}{2\mu}\int_r^{R_0}(R_0^2 r - r^3)\, dr = \frac{\gamma i \pi}{2\mu}\left[R_0^2\frac{r^2}{2} - \frac{r^4}{4}\right]_0^{R_0}$$

$$Q = \frac{\gamma i \pi}{8\mu} R_0^4$$

The quantity of the water transported depends on R^4 so the sized of the pipes matters a lot

The average velocity: $\bar{V} = \frac{Q}{A} = \frac{\gamma i}{8\mu} R_0^2 = \bar{V} = \frac{\gamma i}{32\mu} D^2$ where D is the diameter

Headlosses

We defined the headlosses as the *loss of energy (H) due to* ***stresses.*** To derive the headloss formulation, we look at how much energy we lose as fluids move through the pipe:

We define:

$$j = -\frac{\partial H}{\partial x}$$

The parameter j tells how much head (energy) we lose as we move along the pipe in direction x

$$= -\frac{\partial}{\partial x}\left(\frac{P}{\gamma} + z + \frac{V^2}{2g}\right)$$

0 because V is constant in pipes and does not depend on x

$$= -\frac{\partial}{\partial x}\left(\frac{P}{\gamma} + z\right) = i$$

For constant diameter (D) and velocity (V), namely cylindrical pipes that do not change shape/size, $\boldsymbol{i = j}$, the loss rate is the same as the driver of the flow.

(remember $\bar{V} = \frac{\gamma i}{32\mu} D^2$)

$$j = i = \frac{32\, V\mu}{\gamma D^2}$$

Headlosses

J is an energy loss per unit distance ($\partial/\partial x$) so the **total headloss** is that rate time the total length traveled by the fluid in the pipe:

$$\boldsymbol{h_L = j * L}$$

So for the laminar case:

$$h_L = j\ L = \frac{32\ V\mu}{\gamma D^2} L$$

Headlosses

In general, the energy loss rate (and thus the headloss) are expressed though the *Darcy-Weisbach equation*:

$$\boldsymbol{j = \frac{f}{D}\frac{V^2}{2g}}$$

This equation is applicable to both laminar and turbulent flows but you need to know *the friction factor* $\boldsymbol{f}$ for each specific case

For laminar flow:

$$j = \frac{32\,V\mu}{\gamma D^2} = \frac{f}{D}\frac{V^2}{2g}$$

$$f = \frac{64\mu g}{\rho g D V}$$

and $\frac{\mu}{\rho} = \nu$

$$f = \frac{64}{Re}$$

For laminar flows the friction factor can be found analytically. It's only function of the flow (through Re) and not at all of the pipe itself.

For turbulent flow, the derivation is more complicated, and it's determined experimentally.

Headlosses

Headlosses are inevitably associated to the shear stress at the wall. Moving water causes friction on the wall and that's the source of shear stress, hence the source of losses $h_L = jL$.

So we can also show/define the relationship between the shear stress and the energy loss:

Remember for uniform flow i=j

$$\tau = \gamma i \frac{r}{2} = \gamma j \frac{D}{4} = \rho g \frac{f}{D} \frac{V^2}{2g} \frac{D}{4}$$

$$\tau = \rho \frac{f}{8} V^2$$

We can also introduce another parameter that is often used: *the friction velocity* $v_* = \sqrt{\frac{\tau}{\rho}}$

$$v_* = \sqrt{\frac{f}{8}}\, V \quad \longrightarrow \quad \frac{v_*}{V} = \sqrt{\frac{f}{8}}$$

Note: *v** is not an actual velocity of the fluid—it is merely a quantity that has dimensions of velocity.

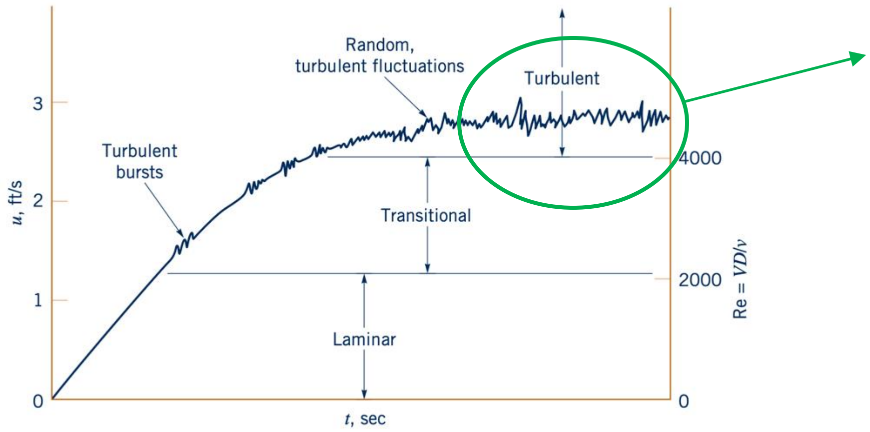
Turbulent Flows ($Re >>>$)

At the beginning, we saw that the transition between laminar (smooth) and turbulent (chaotic) flows depends on the value of the dimensionless Reynolds number (Re).

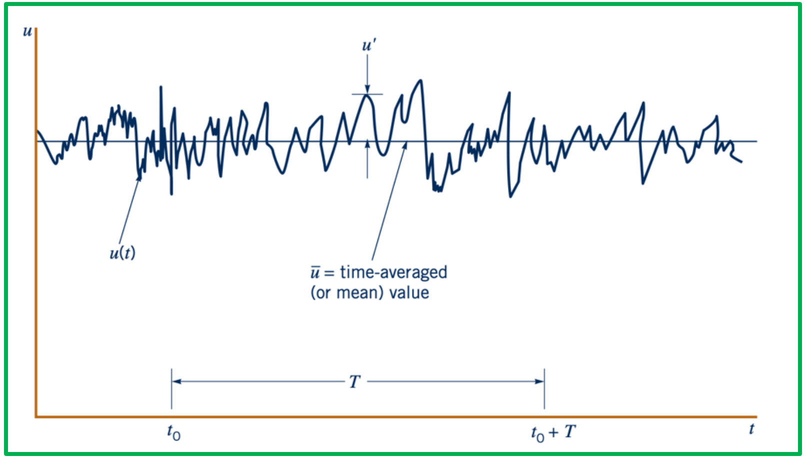
As general rule for pipe flow:

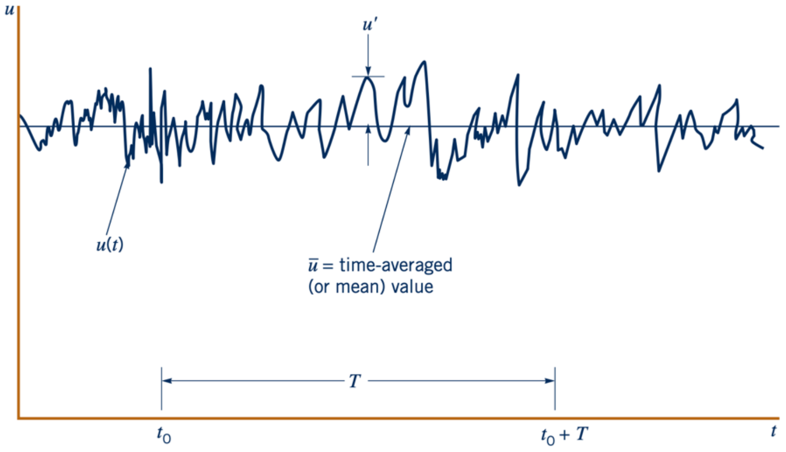
- Re < 2100 → **LAMINAR**
- Re> 4000 → **TURBULENT**

If we start from quiescent water and then we slowly increase the flow, we'll see something like this:



If we measure the velocity in a point of the pipe, we'll see a jumpy signal like:





Turbulent Flows are ***irregular*** and ***random*** in nature.

Turbulence enhance mixing which is very important for many physical processes. Without turbulence it would be virtually impossible to carry out life as we now know it.

Randomness → velocity $u = u(x, y, z, t)$ jumps a around a mean value:

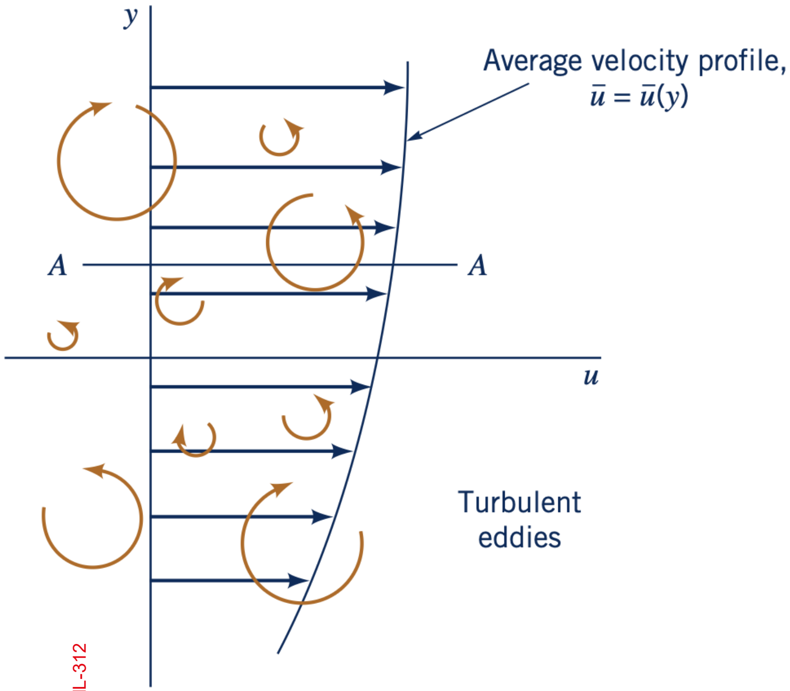
$$\bar{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(x, y, z, t)\, dt$$

→ the velocity and its three components can be expresses as the sum between it's mean value $\bar{u}$ and the fluctuations u' at any instant in time.

$$u = \bar{u} + u' \qquad \text{or} \qquad u' = u - \bar{u}$$

Several experiments and theories have shown that for turbulent flows, $\tau \neq \mu \frac{d\bar{u}}{dy}$!

So what are the turbulent stresses??
The physical explanation lies in the mechanism that produces the stresses in turbulent flows:

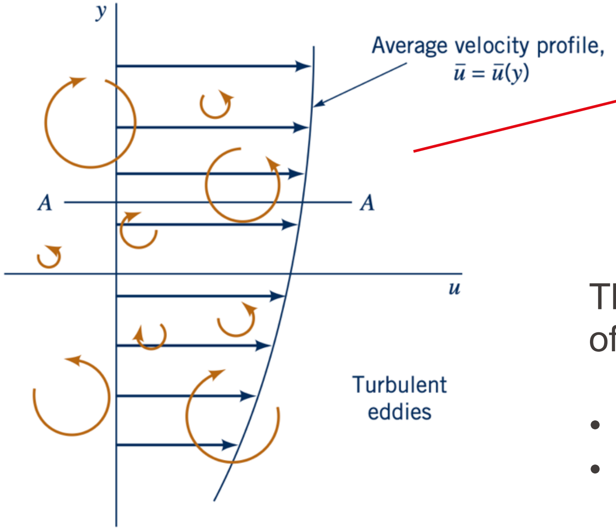


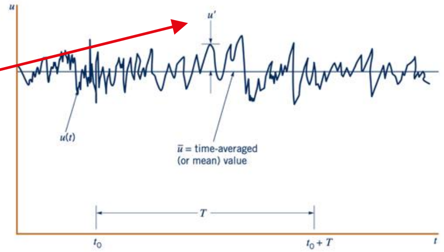
- Series of random, 3-dimensional eddies (vortices) of many different sizes

- Eddies moves big parts of fluids in the space, moving slow-moving parts (slow-momentum) into fast-moving zones (high-momentum) and vice versa. Basically, eddies mix momentum in the flow.

- This contrast creates drag (of either slow fluid slowing down high fluid or fast fluid accelerating slow fluid). **This drag create shear stress**.

- Because big finite parts of fluid are transported, the shear force is large, much larger than the viscous.





The amount of this momentum transfer is proportional to the size of the eddies which are related to the the velocity fluctuations:

- u′ → for the x component of velocity
- v′ → for the rate of mass transfer crossing the plane A-A

Turbulent shear stress (also known as the Reynolds stresses in honor of Osbourne Reynolds who first discussed them in 1895) and are found to be positive.

$$\tau_{turb} = -\rho\overline{u'v'}$$

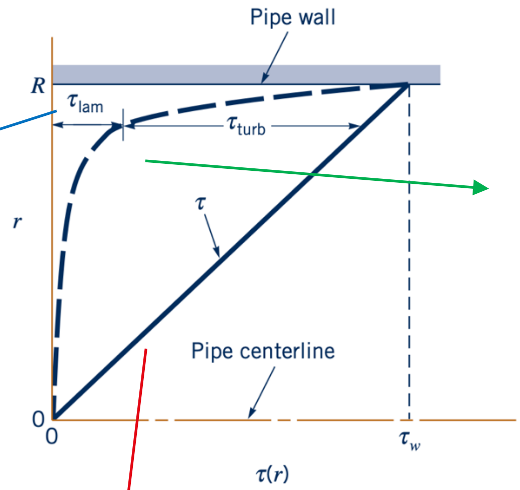
So the **total stress** can be seen as:

$$\tau_T = \mu\frac{du}{dy} - \rho\overline{u'v'} = \tau_{lam} + \tau_{turb}$$

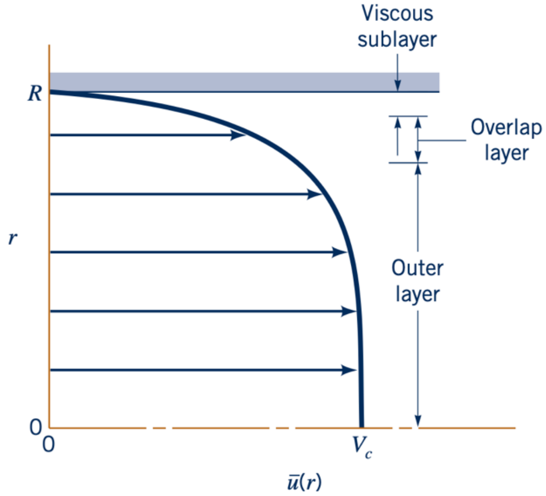
What is the relative magnitude of τ_{lam} compared to τ_{turb} ? (dash-line below)

Typical measurements have shown this relative distribution:

Near the wall (i.e., the Viscous Sublayer), the laminar shear stress is dominant



Away from the wall (i.e., the turbulent portion of shear stress is dominant

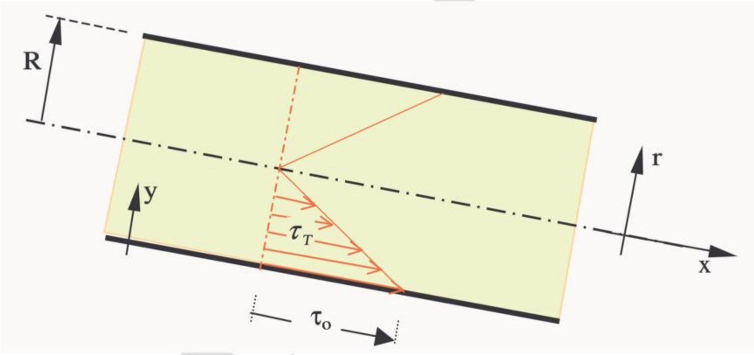


Total shear stress proportional to the distance from the centerline of the pipe, as we've seen earlier (triangular distribution)

Corresponding typical velocity profile (flatter than the viscous parabolic one because turbulence redistributes momentum)

It is easy to see that deriving a velocity distribution for turbulent flows is trickier because the relation between velocity and stresses is not obvious.

For classical engineering applications we rely on empirical evidences that at least are almost universal

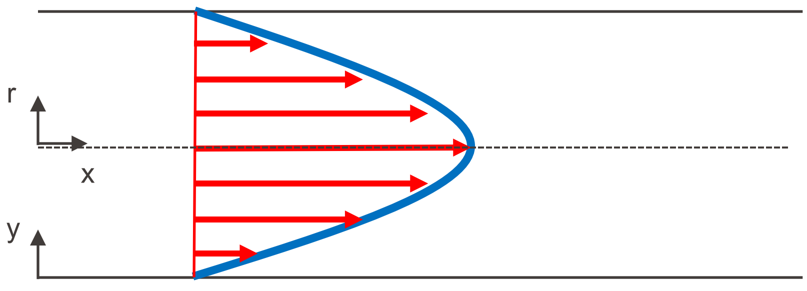


$$\tau_T = \mu \frac{du}{dy} - \rho \overline{u'v'} = \tau_{lam} + \tau_{turb}$$

- So far we showed that the stresses have a triangular distribution and are max at the wall τ_0
- **The stress at the wall is the main cause of headlosses**
- We define a term associated to τ_0 that is important and comes out in the velocity formulas we found experimentally, the *friction velocity*

$$v_* = \sqrt{\frac{\tau_0}{\rho}}$$

For laminar flows, we said that the velocity distribution is parabolic, and we found the friction factor and headloss expressions analytically

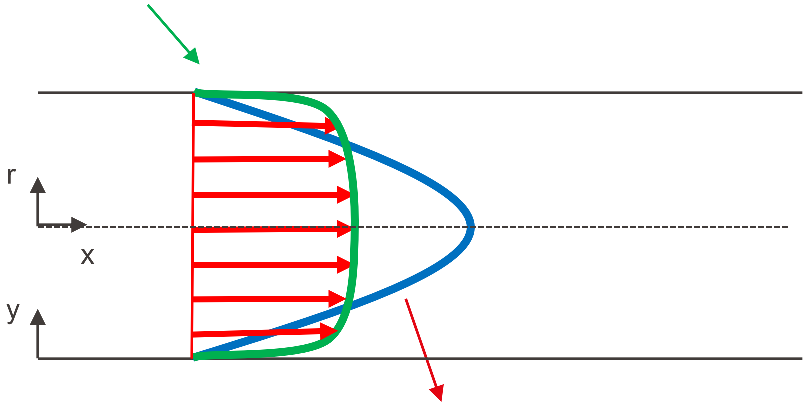


$$V_x = \frac{\gamma i}{4\mu}(R_0^2 - r^2)$$

$$f = \frac{64}{Re}$$

Note that is independent from the material or condition of the pipe!

For turbulent flows, expression was found experimentally **as a function of the friction velocity**!



Laminar for comparison

$$V_x = \frac{v_*}{k} ln\left(\frac{y}{y_0}\right)$$

$$f = ??$$

Where $k \approx 0.4$ is the Von Karman constant and y_0 is an important parameter **related to the roughness of the pipe** which heavily influences the value of the friction factor f

Note: y is the coordinate from the wall, also $y = R_0 - r$

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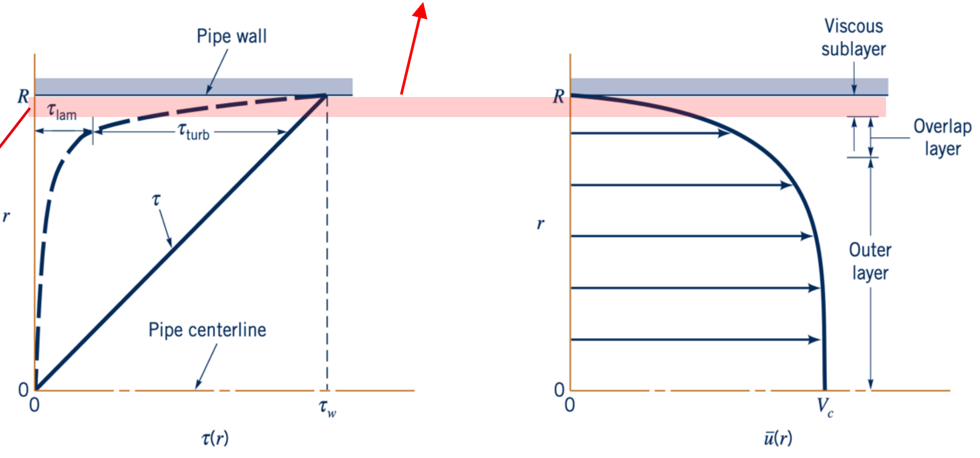
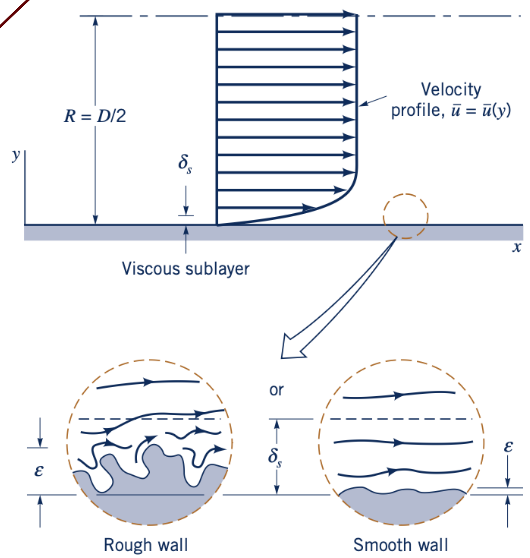
Viscous sublayer where turbulence ~0

$\delta_s \cong 10\nu/v_*$

So, to understand the friction, we need to understand y_0, and to understand y_0, we need to understand what happens near the wall!



Therefore, it depends on how the pipe is made! **So we define different type of pipe wall roughness ϵ !**

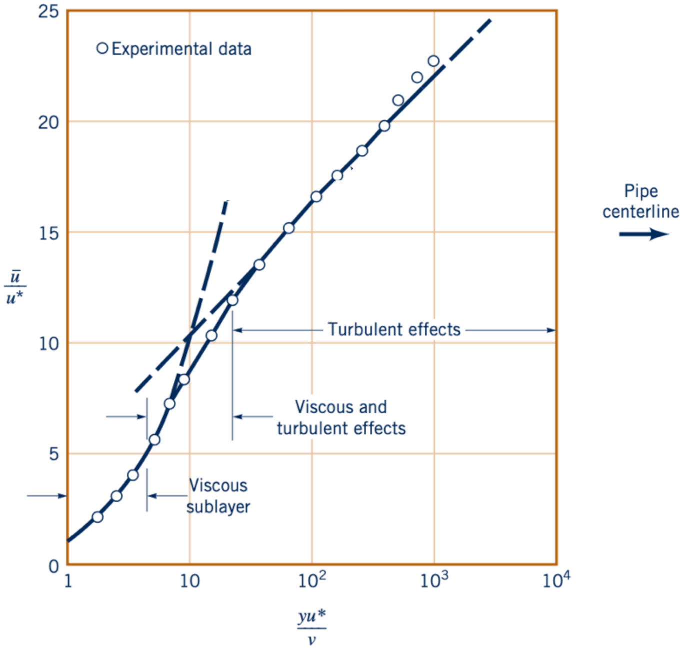


- Hydraulically **Smooth** wall $\Rightarrow \epsilon < 5\,\nu/v_* \;\left(< \frac{\delta_s}{2}\right)$
- Hydraulically **Rough** wall $\Rightarrow \epsilon > 70\,\nu/v_*$

❑ **Hydraulically Smooth wall** $\Rightarrow \epsilon < 5\,\nu/v_*$

Velocity distribution:

$$\begin{cases} \dfrac{V}{v_*} = \dfrac{v_* y}{\nu} & for\ 0 < \dfrac{v_* y}{\nu} < 5 \\ \dfrac{V}{v_*} = 2.5\ln\left(\dfrac{y v_*}{\nu}\right) + 5 & for\ 20 < \dfrac{v_* y}{\nu} < 10^5 \end{cases}$$



Friction factor: $$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{Re\sqrt{f}}{2.51}\right) \qquad for\ Re > 3000$$

For hydraulically smooth wall situations, the velocity profiles and the friction factor do not depend on the physical roughness of the pipe

❑ Hydraulically Rough wall $\Rightarrow \epsilon < 70\, \nu/v_*$

In this case, the roughness size is so big (much bigger than viscous sublayer) that the viscous sublayer disappear along with the viscous effects on the flow. The flow becomes **fully turbulent** and y_0 depends completely by the physical size of the roughness ϵ :

$$\frac{V}{v_*} = 2.5 \ln\left(\frac{y}{\epsilon}\right) + 8.5$$

$\boldsymbol{\epsilon}$ **sometimes is also written as** $\boldsymbol{k_s}$ which was the size of the sand used by Johann Nikuradse (student of Prandtl) who measured the flow in pipes by systematically changing the roughness gluing sand of different size inside commercially available pipes.

So now friction is only a function of the roughness size (or better the relative roughness as compared to the diameter ϵ/D) and not the size of the viscous sublayer

Friction factor:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{1}{3.71}\frac{\epsilon}{D}\right)$$

To recap

- Turbulent Flows:
    - Smooth walls $$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.51}{Re\sqrt{f}}\right)$$
    - Rough walls $$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{1}{3.71}\frac{\epsilon}{D}\right)$$

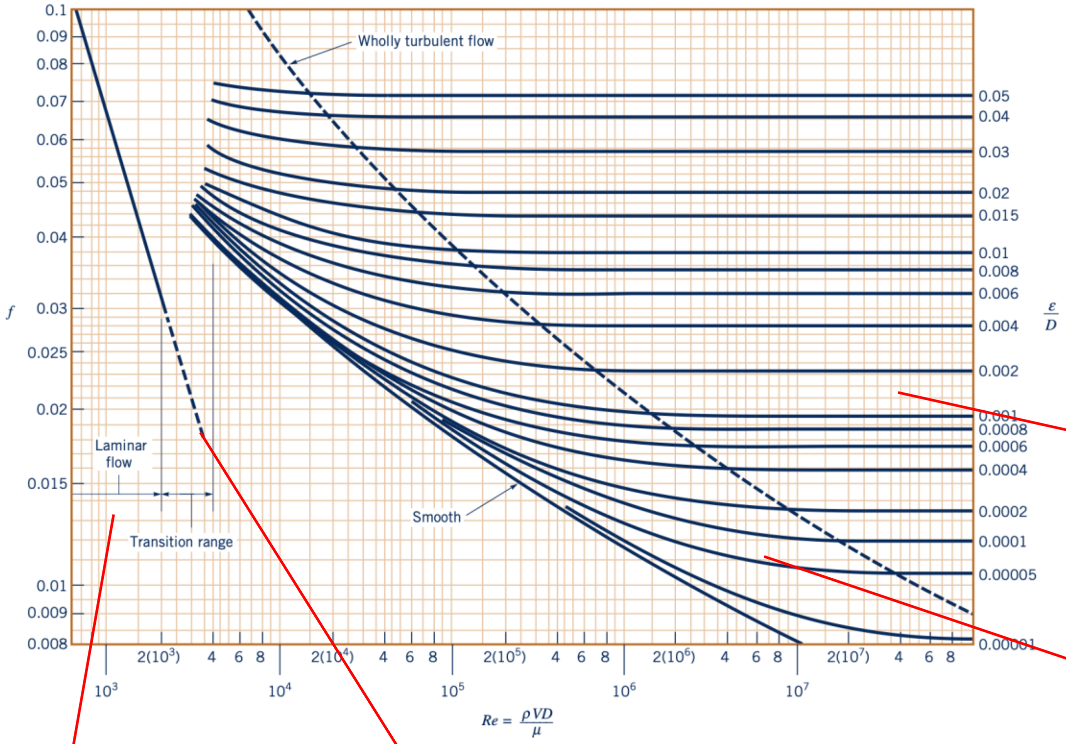
Colebrook & White (1939) proposed the following semi empirical formula:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{1}{3.71}\frac{\epsilon}{D} + \frac{2.51}{Re\sqrt{f}}\right)$$

Few notes:

- For small values of Re, we're in smooth wall and the second term inside the parenthesis matters most
- For high values of Re the second matters less, we're in rough wall and the friction factor only depends on the relative roughness
- It's implicit! The friction factor is on both sides of the equation so to solve for it we'll have to iterate!
- This formula was tested on *commercial* pipes (i.e., that you can buy for your project) and applies in all the non-laminar regimes of the *Moody diagram* (next slide)

The Moody diagram



- Laminar Flows: $f = \frac{64}{Re}$
- Turbulent Flows: $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{1}{3.71}\frac{\epsilon_S}{D} + \frac{2.51}{Re\sqrt{f}}\right)$

Different lines for different relative roughness (i.e., different material and/or wear of pipes, normalized by diameter)

Completely (or *wholly*) turbulent flows, the roughness dominates the character of the flow near the wall, so friction only depends on the relative roughness

In the region between “smooth wall” line and wholly turbulent, the friction factor depends on both Re and relative roughness, so we have to use the whole C&W equation. Note: this happens often.

Laminar flow, resistance is independent form relative roughness

In this gap, the flow is in transition between laminar and turbulent (it can be laminar, turbulent or an unsteady mix of both). Here we can’t really determine a friction factor.

Minor Losses

So far we've seen that **Major Headlosses** along the pipe are the product of the friction factor times the length of the pipe:

$$h_L = J \cdot L = \frac{f}{D}\frac{V^2}{2g}L$$

There are additional minor losses ($h_{L_{minor}}$) due to other components of the pipeline (valves, bends, tees, etc.) which add to the overall head loss of the system.

Typically:

$$h_{L_{minor}} = K_L \frac{V^2}{2g}$$

where K_L depends on the geometry of the component!

- Flow entrance from a reservoir to a pipes has minor losses. Each entrance geometry has an associated coefficient. For sharp edges, K_L = 0.5 for example
- Flow exit from a pipe to a reservoir has losses with K_L = 0.5
- Gradual/sudden contractions and enlargement have different coefficients
- Bends

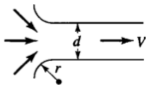
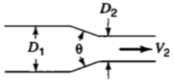
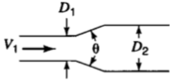
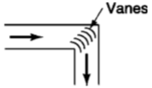
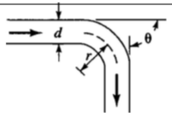
(See table in next slide)

Table 4.3.1 Minor Loss Coefficients for Pipe Flow

Type of minor loss	K Loss in terms of $V^2/2g$
Pipe fittings:	
90° elbow, regular	0.21–0.30
90° elbow, long radius	0.14–0.23
45° elbow, regular	0.2
Return bend, regular	0.4
Return bend, long radius	0.3
AWWA tee, flow through side outlet	0.5–1.80
AWWA tee, flow through run	0.1–0.6
AWWA tee, flow split side inlet to run	0.5–1.8
Valves:	
Butterfly valve (θ = 90° for closed valve)*	
θ = 0°	0.3–1.3
θ = 10°	0.46–0.52
θ = 20°	1.38–1.54
θ = 30°	3.6–3.9
θ = 40°	10–11
θ = 50°	31–33
θ = 60°	90–120
Check valves (swing check) fully open	0.6–2.5
Gate valves (4 to 12 in) fully open	0.07–0.14
1/4 closed	0.47–0.55
1/2 closed	2.2–2.6
3/4 closed	12–16
Sluice gates:	
As submerged port in 12-in wall	0.8
As contraction in conduit	0.5
Width equal to conduit width and without top submergence	0.2
Entrance and exit losses:	
Entrance, bellmouthed	0.04
Entrance, slightly taunted	0.23
Entrance, square edged	0.5
Entrance, projecting	1.0
Exit, bellmouthed	$0.1\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)$
Exit, submerged pipe to still water	1.0

*Loss coefficients for partially open conditions may vary widely. Individual manufacturers should be consulted for specific conditions.

Table 4.3.2 Loss Coefficients for Various Transitions and Fittings

Description	Sketch	Additional data	K		Source
Pipe entrance		r/d	K_e		(a)
$h_{l_m}=K_e V^2/2g$		0.0	0.50		
		0.1	0.12		
		>0.2	0.03		
Contraction		D_2/D_1	K_C θ = 60°	K_C θ = 180°	(a)
$h_{l_m}=K_C V_2^2/2g$		0.0	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.32	
		0.80	0.05	0.18	
		0.90	0.04	0.10	
Expansion		D_1/D_2	K_E θ = 10°	K_E θ = 180°	(a)
$h_{l_m}=K_E V_1^2/2g$		0.0		1.00	
		0.20	0.13	0.92	
		0.40	0.11	0.72	
		0.60	0.06	0.42	
		0.80	0.03	0.16	
90° miter bend		Without vanes	$K_b = 1.1$		(b)
		With vanes	$K_b = 0.2$		(b)
Smooth bend		r/d	K_b θ = 45°	K_b θ = 90°	(c) and (d)
		1	0.10	0.35	
		2	0.09	0.19	
		4	0.10	0.16	
		6	0.12	0.21	
Threaded pipe fittings	Globe valve—wide open		$K_v = 10.0$		(b)
	Angle valve—wide open		$K_v = 5.0$		
	Gate valve—wide open		$K_v = 0.2$		
	Gate valve—half open		$K_v = 5.6$		
	Return bend		$K_b = 2.2$		
	Tee		$K_t = 1.8$		
	90° elbow		$K_b = 0.9$		
	45° elbow		$K_b = 0.4$		

(a) ASHRAE (1977)
(b) Streeter (1961)
(c) Beij (1938)
(d) Idel'chik (1966)

Source: After Roberson et al. (1988).

From Mays, L.W., 2019. Water Resources Engineering, 3rd Edition. John Wiley & Sons.

EPFL

Different categories of problems when designing pipe flows

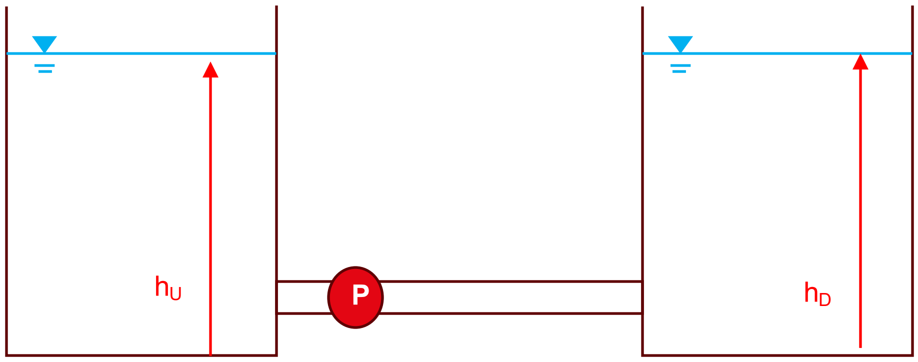
Three categories of problems can be identified for developed turbulent flow in a pipe of length L:

Category	*Known*	*Unknown*
1	Q, D, e, ν	h_L
2	D, e, ν, h_L	Q
3	Q, e, ν, h_L	D

A category 1 problem is straightforward and requires no iteration procedure when using the Moody diagram.

Category 2 and 3 problems are more like problems encountered in engineering design situations and require an iterative trial-and-error process when using the Moody diagram.

Example



D = 30 cm
L = 1 km
ϵ = 0.6 mm
Q = 180 l/s

P?

Let's take two reservoirs for urban water distribution connected by a pipe of diameter D and length L. The water level in the two reservoirs is the same (h_U=h_D). We need to convey a water discharge Q. To move water, we either have potential energy (i.e., the reservoirs are at two different elevations) or we need to supply energy with a pump. In this scenario the two reservoir are at the same elevation z so we install a pump P.

What is the power needed for the pump?

SOLUTION ON GOODNOTES FILE